QRLG, Statistical Regularization and the Bounce

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Motivation

General Relativity in
Ashtekar variables

$$A^i_a(x), E^a_i(x)$$

Symmetry reduction

Mini SuperSpace

$$c(t), p(t)$$

Midi SuperSpace

$$A^i{}_a(r,t) E_i{}^a(r,t)$$

“LQG derived”
quantization

LQC

LQBH

*Ashtekar, Agullo, Barrau, Bojowald,
Campiglia, Corichi, Gambini, Giesel, Grain, Hofmann,
Henderson, Kaminski, Lewandowski, Mena Marugan,
Modesto, Nelson, Pawlowski, Pullin, Singh, Sloan,
Taveras, Thiemann,, Winkler, Wilson-Ewing*

Motivation

General Relativity in
Ashtekar variables

$$A_a^i(x), E_i^a(x)$$

“LQG” quantization

LQG

Symmetry reduction:

Quantum Reduced LG Program

- Symmetry reduced models have “smart” frames: systems of coordinates adapted to the symmetries
- In these coordinate systems the imposition of the symmetries allows to further simplify the form of the metric and Einstein Equations

Symmetry reduction in two steps:

1) Gauge fix the metric (without symmetry reduction):
Study the second class constraint system: Reduced Phase Space

A. Dirac Brackets
B. Gauge Unfixing Mitra, Rajaraman, Anishetty, Vytheeswaran:
   - Ordinary Poisson Brackets for the non gauge fixed variables
   - Modified Constraints to preserve the gauge fixing during the evolution

Bodendorfer, Thiemann, Thurn- Bodendorfer, Duch, Lewandowski, Świeżewski

2) Implement the symmetry reduction in the reduced phase space

Quantization:

A. Quantize Dirac Brackets
B. Quantize the classically Reduced Phase Space (with or without symmetry)

   Bodendorfer Diagonal gauge *Bodendorfer, Duch, Lewandowski, Świeżewski, Zipfel* Radial gauge

QRLG Quantization:

1. Impose the second class constraints weakly in the Full Hilbert Space:
   Selects the reduced states i.e. the quantum reduced phase space

2. Study the modified dynamics (preserving the gauge fixing) projecting the constraints
3. Impose the symmetry reduction on the reduced states using coherent states

Program:

Find quantum symmetry reduction compatible with given metrics

- Cosmological metrics: diagonal gauge fixing
- Black Holes: radial gauge fixing

Quantum Reduced LG: Cosmology

Look at the inhomogeneous line element
in the BKL conjecture Belinski-Khalatnikov-Lifshitz '70 :

$$ds^2 = N^2(t)dt^2 - e^{2\alpha(t,x)}(e^{2\beta(t,x)})_{ij}\,\omega^i \otimes \omega^j$$

α Describes the Volume

β (diagonal matrix, Tr $\beta = 0$)
Describes local anisotropies

ω one forms corresponding to an homogeneous Bianchi model

GOAL:
find a quantum symmetry reduction of LQG compatible with this line element

If we remove the spatial dependence from α and β, we can recover generic Bianchi models

How to implement the reduction on the holonomies and consistently impose $\chi_i=0$?

Strategy: Mimic the spinfoam procedure

Engle, Pereira, Rovelli,Livine '07- '08

Use a Projector P_χ from the full LQG Hilbert space to the reduced states and project operators and constraints

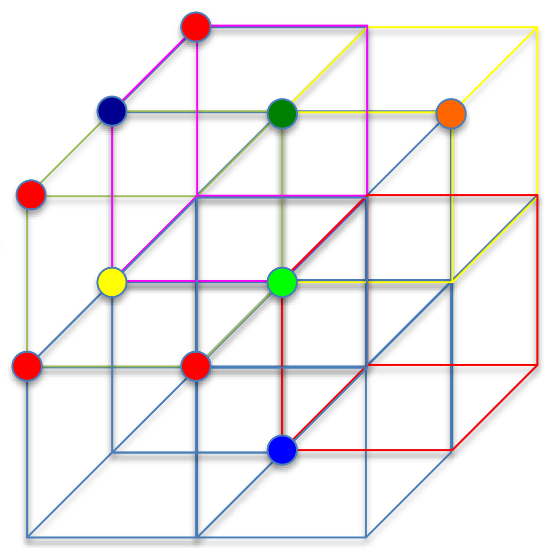
$\hat{G}_i(A,E) \Rightarrow P_\chi^\dagger \hat{G}_i P_\chi$ reduced intertwiners

$\hat{V}_a(A,E) \Rightarrow P_\chi^\dagger \hat{V}_a P_\chi$ s-knot states

Ashtekar, Lewandowski, Marolf, Mourao, Thiemann

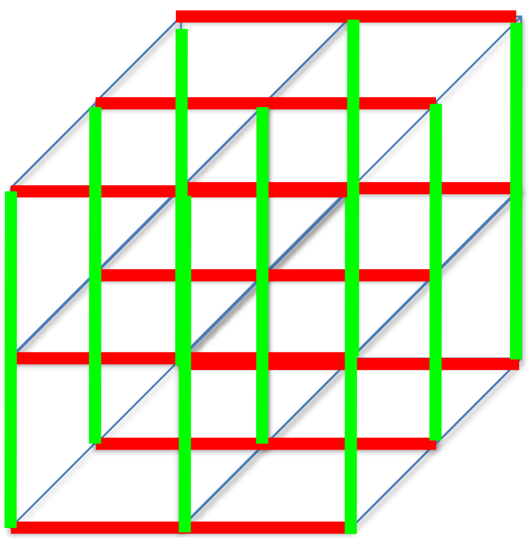
The Inhomogenous sector

Different
Reduced SU(2)
intertwiners:
inhomogeneities

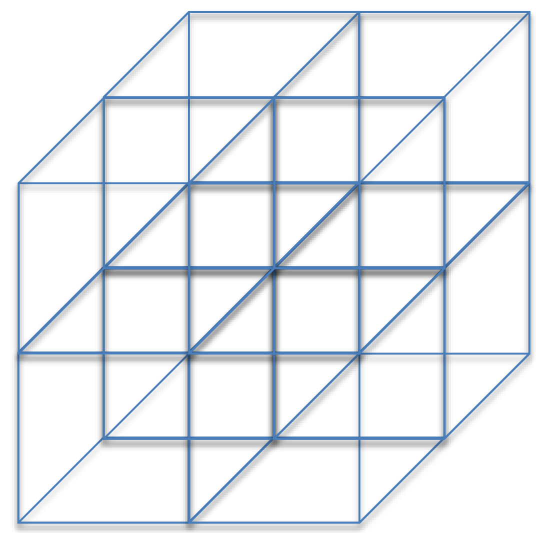


U(1)
Spin labels:
Anisotropies

Homogeneous and anisotropic sector



Homogeneous and Isotropic sector



Loop Quantum Gravity in diagonal triad gauge ?

Kinematically Yes *Alesci, Cianfrani, Rovelli*

but non trivial Hamiltonian

(the evolution may not preserve the gauge; in the BKL hypotesis it does).

Study the Hamiltonian à la Thiemann ('96-'98) using operators defined in the reduced Hilbert space on coherent states

Hall, Thiemann, Winkler, Sahlmann, Bahr

Large distance asymptotic behaviour Bianchi Magliaro Perini

$$\Psi_{H_l}(h_l) \simeq \sum_{j_l, i_n} \prod_l e^{-\frac{(j_l - j_l^0)^2}{2\sigma_l^2}} e^{-i\xi_l j_l} \Big(\prod_n \Phi_{i_n}\Big) \Psi_{j_l, i_n}(h_l)$$

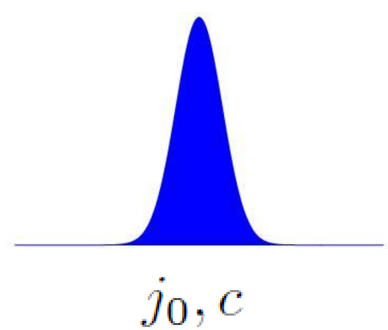
Codes the intrinsic geometry

Codes the extrinsic curvature

Livine-Speziale Intertwiners

$$j_0 = \frac{|E|}{8\pi G\hbar\gamma}$$

$$\xi \sim K = c$$



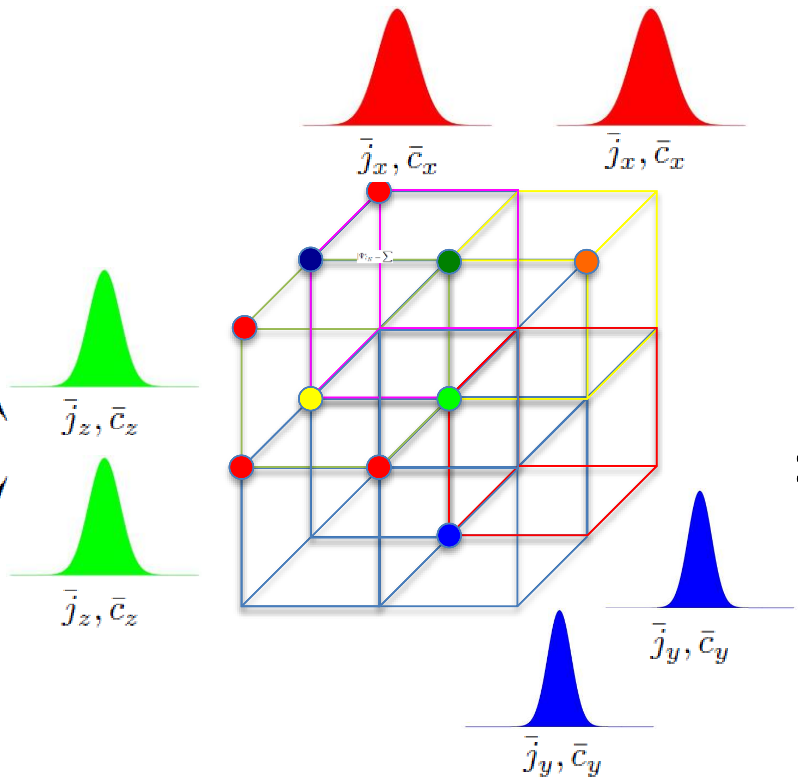
Project the coherent states in the reduced Hilbert space

Collective Coherent state

$$N = N_x N_y N_z$$

Total number of nodes

$$|\Psi\rangle_N = \sum$$



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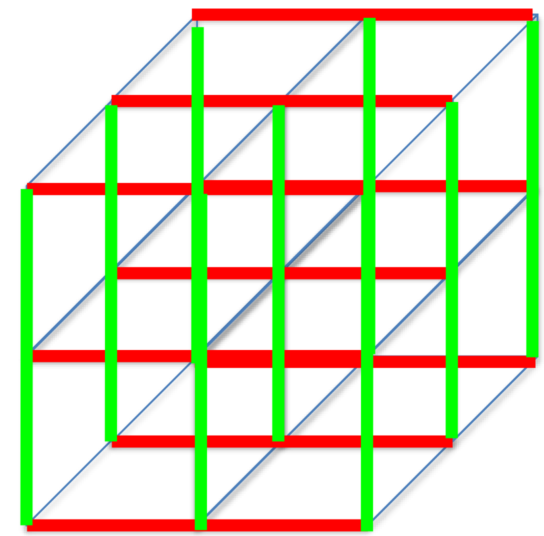


$$P_x = N_y N_z p_x \qquad P_y = N_z N_x p_y \qquad P_z = N_x N_y p_z$$

$$C_x = N_x c_x \qquad C_y = N_y c_y \qquad C_z = N_z c_z$$

Collective Variables

Comparison with LQC

$$H = \frac{2}{\gamma^2}\mathcal{N}\Bigg(\sqrt{\frac{p^x p^y}{p^z}}\,\frac{\sin\mu_x c_x}{\mu_x}\,\frac{\sin\mu_y c_y}{\mu_y} + \sqrt{\frac{p^y p^z}{p^x}}\,\frac{\sin\mu_y c_y}{\mu_y}\,\frac{\sin\mu_z c_z}{\mu_z} + \sqrt{\frac{p^z p^x}{p^y}}\,\frac{\sin\mu_z c_z}{\mu_z}\,\frac{\sin\mu_x c_x}{\mu_x}\Bigg)$$

$$\langle {}^R\hat{H}^{1/2}\rangle_N \approx \frac{2}{\gamma^2}\mathcal{N}\Bigg(N_x N_y\sqrt{\frac{P^x\,P^y}{P^z}}\,\sin\frac{C_x}{N_x}\sin\frac{C_y}{N_y} + N_y N_z\sqrt{\frac{P^y\,P^z}{P^x}}\,\sin\frac{C_y}{N_y}\sin\frac{C_z}{N_z} + N_z N_x\sqrt{\frac{P^z\,P^x}{P^y}}\,\sin\frac{C_z}{N_z}\sin\frac{C_x}{N_x}\Bigg)$$

$$\bar{\mu}_x\bar{\mu}_y = \frac{\Delta l_P^2}{p^z}\,,\quad \bar{\mu}_y\bar{\mu}_z = \frac{\Delta l_P^2}{p^x}\,,\quad \bar{\mu}_z\bar{\mu}_x = \frac{\Delta l_P^2}{p^y}$$

$$\frac{1}{N_x}\frac{1}{N_y} = \frac{8\pi\gamma l_P^2}{P^z}j_z\,,\quad \frac{1}{N_y}\frac{1}{N_z} = \frac{8\pi\gamma l_P^2}{P^x}j_x\,,\quad \frac{1}{N_z}\frac{1}{N_x} = \frac{8\pi\gamma l_P^2}{P^y}j_y$$

Ashtekar, Wilson-Ewing

$$\mu_i = \frac{1}{N_i}$$

Volume corrections

$$\frac{1}{\sqrt{\bar{p}_i}} \to \frac{1}{\sqrt{\bar{p}_i}}\left[1 + \left(\frac{\pi\gamma l_P^2}{\bar{p}_i}\right)^2\right]$$

$$\frac{1}{\sqrt{\bar{P}_i}} \to \frac{1}{\sqrt{\bar{P}_i}}\left[1 + \frac{N^2}{N_i^2}\left(\frac{\pi\gamma l_P^2}{\bar{P}_i}\right)^2\right]$$

Improved LQC from QRLG

Homogeneous case

$$p = 8\pi\gamma\ell_P N^{2/3} j \qquad c = N^{1/3}\theta$$

Collective variables

$$\langle N, j, \theta | \hat{H}^{grav} | N, j, \theta \rangle = \frac{3}{8\pi G\gamma^2} \sqrt{p} N^{2/3} \sin^2(N^{-1/3} c)$$

Collective coherent states

FIXED N

$$\mu_0 = N^{-1/3}$$

Is there a way to reproduce the improved scheme ?

(without a graph changing H)

Statistical approach: density matrix

Count microstates (N,j,θ) compatible with macroscopic configuration (c,p)

Few big or many small ? Both !

$$\rho_{p,c} = \sum_N c_N \, |N, j(p,N), \theta(c,N)\rangle\langle N, j(p,N), \theta(c,N)|$$

Key Observation: for fixed area, j has a minimum → N has a maximum

$$\boxed{p = 8\pi\gamma\ell_P^2 N_{max}^{2/3} j_0}$$

$$c_N = \frac{1}{2^{N_{max}}}\binom{N_{max}}{N} \qquad Tr(\rho_{p,c}H) = \frac{3}{8\pi G\gamma^2}\sqrt{p}\, f(c, N_{max})$$

Approximating the binomial with a Gaussian and
performing the sum with a saddle point approximation

$$f(c, N_{max}) \sim \frac{\sin^2(\tilde{\mu}c)}{\tilde{\mu}^2} \qquad \tilde{\mu} = (N_{max}/2)^{-1/3}$$

$$p\tilde{\mu}^2 = \tilde{\Delta} \qquad \tilde{\Delta} = (2)^{2/3} 8\pi\gamma\ell_{pl}^2 j_0$$

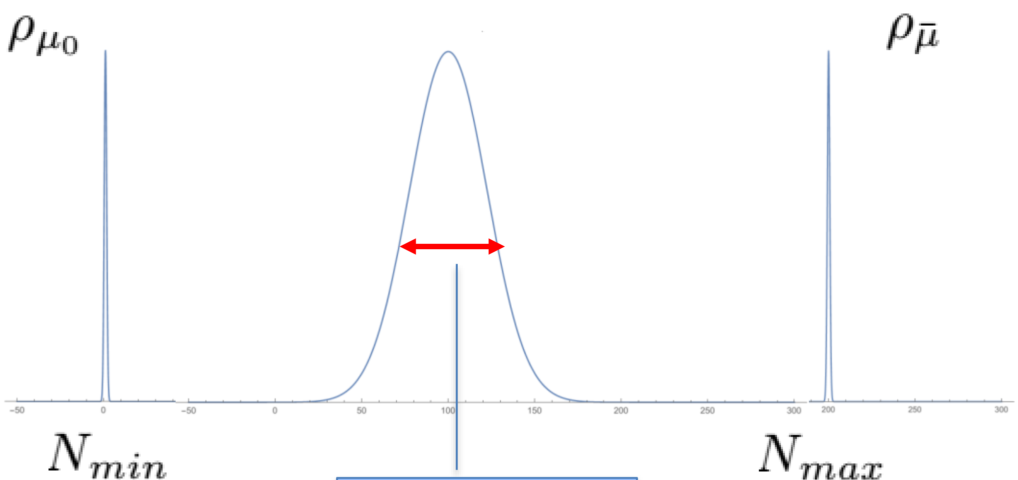
Improved scheme from QRLG

Statistical Regularization

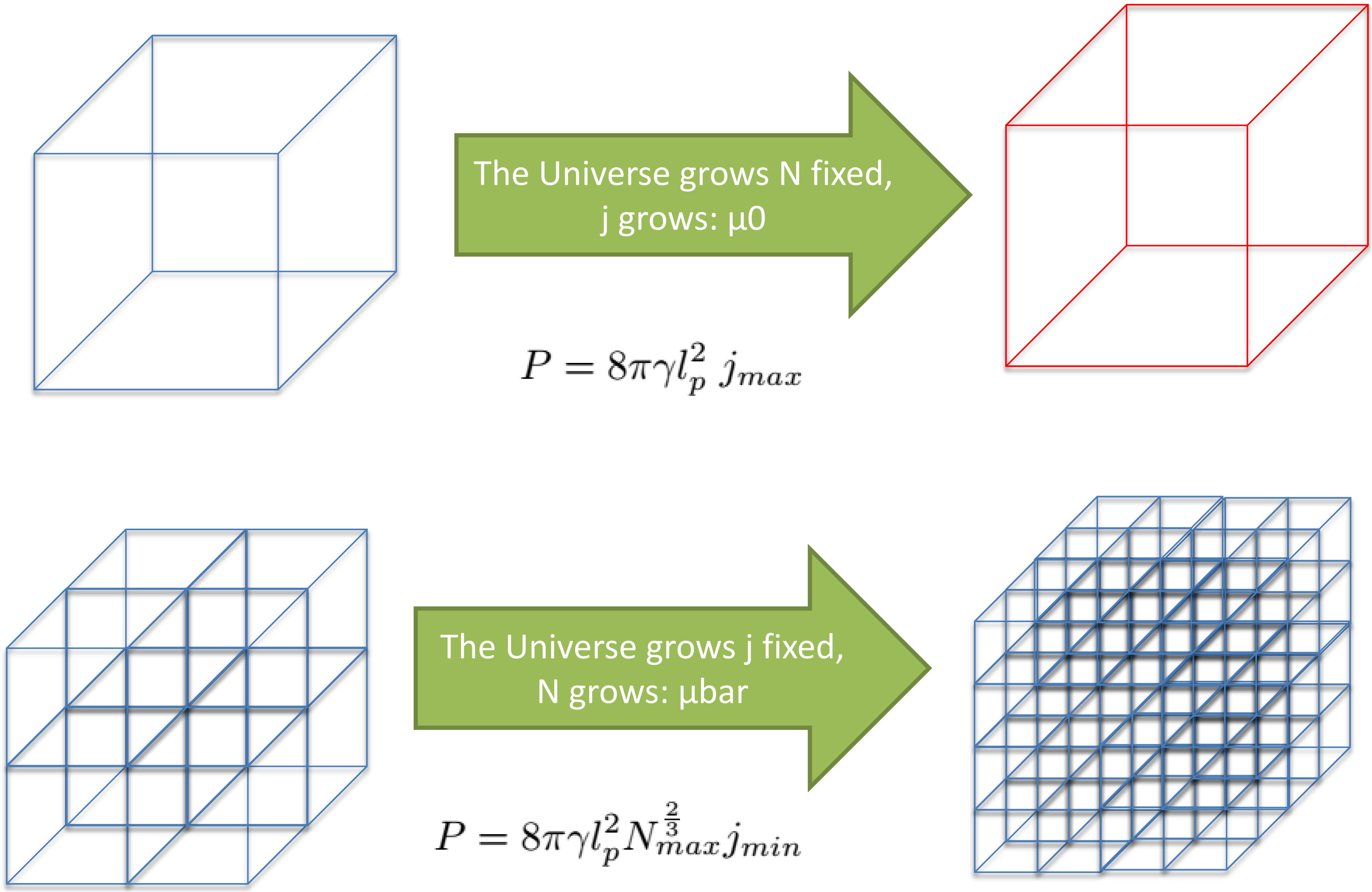
$\rho(N)$ Density distribution of the graphs:
Fixes the scheme and the Hamiltonian

EA Botta Stagno
To appear

$$H_{QRLG}(\rho, C, P) = \int dN \rho(N) \sqrt{P(N)} \frac{\sin^2(\mu(N)C)}{\mu^2(N)} \qquad \mu(N) = \frac{1}{N^{1/3}} \qquad P = 8\pi\gamma l_p^2 N^{\frac{2}{3}} j$$



SPREAD !!!


The Universe grows N fixed,
j grows: µ0
$P = 8\pi\gamma l_p^2\ j_{max}$
The Universe grows j fixed,
N grows: µbar
$P = 8\pi\gamma l_p^2 N_{max}^{\frac{2}{3}} j_{min}$

Corrections

EA Botta Cianfrani Liberati

b, v, variables

$$b = \frac{c}{p^{1/2}} \qquad v = \frac{p^{3/2}}{2\pi\gamma}$$

$$\{v, b\} = -2$$

$$V = 2\pi\gamma v \qquad b = \gamma H \qquad \frac{\dot{a}}{a} = \frac{\dot{v}}{3v}$$

$$\boxed{H_{l.q.c} = -\frac{3v}{4\Delta\gamma} sin^2(\sqrt{\Delta} b) + \frac{P_\phi^2}{4\pi\gamma v}}$$

$$\begin{cases} \dot{v} = \dfrac{3v}{2\sqrt{\Delta}\gamma} sin(2\sqrt{\Delta} b) \\ \dot{b} = -\dfrac{P_\phi^2}{\pi\gamma\ v^2} \end{cases}$$

$$\rho_m = \frac{P_\phi^2}{2V^2}$$

Energy density (massless scalar field)

$$\rho_{cr} = \frac{3}{8\pi\gamma^2\Delta}$$

Critical energy density

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3}\rho(1 - \frac{\rho}{\rho_{cr}})$$

LQC Friedmann equation

$$\boxed{H_{QRLG} = -\frac{3v}{4\Delta\gamma}\ \sin^2(b\sqrt{\Delta}) + \frac{P_\phi^2}{4\pi\gamma v} - \frac{b^2\Delta^{3/2}}{24\pi\gamma^2}\cos(2b\sqrt{\Delta}),}$$

$$\begin{cases} \dot{v} = \dfrac{3v}{2\sqrt{\Delta}\gamma}\sin(2b\sqrt{\Delta})\left(1 + \dfrac{\Delta^2 b}{9\pi\gamma v}\cot(2b\sqrt{\Delta}) - \dfrac{\Delta^{5/2}b^2}{9\pi\gamma v}\right) \\ \dot{b} = -\dfrac{P_\phi^2}{\pi\gamma\ v^2} + \dfrac{b^2\Delta^{3/2}}{12\pi\gamma^2 v}\cos(2b\sqrt{\Delta})\,, \end{cases}$$

$$\rho_{\mathrm{g}} = -\frac{\Delta^{3/2}b^2}{9V}\,,$$

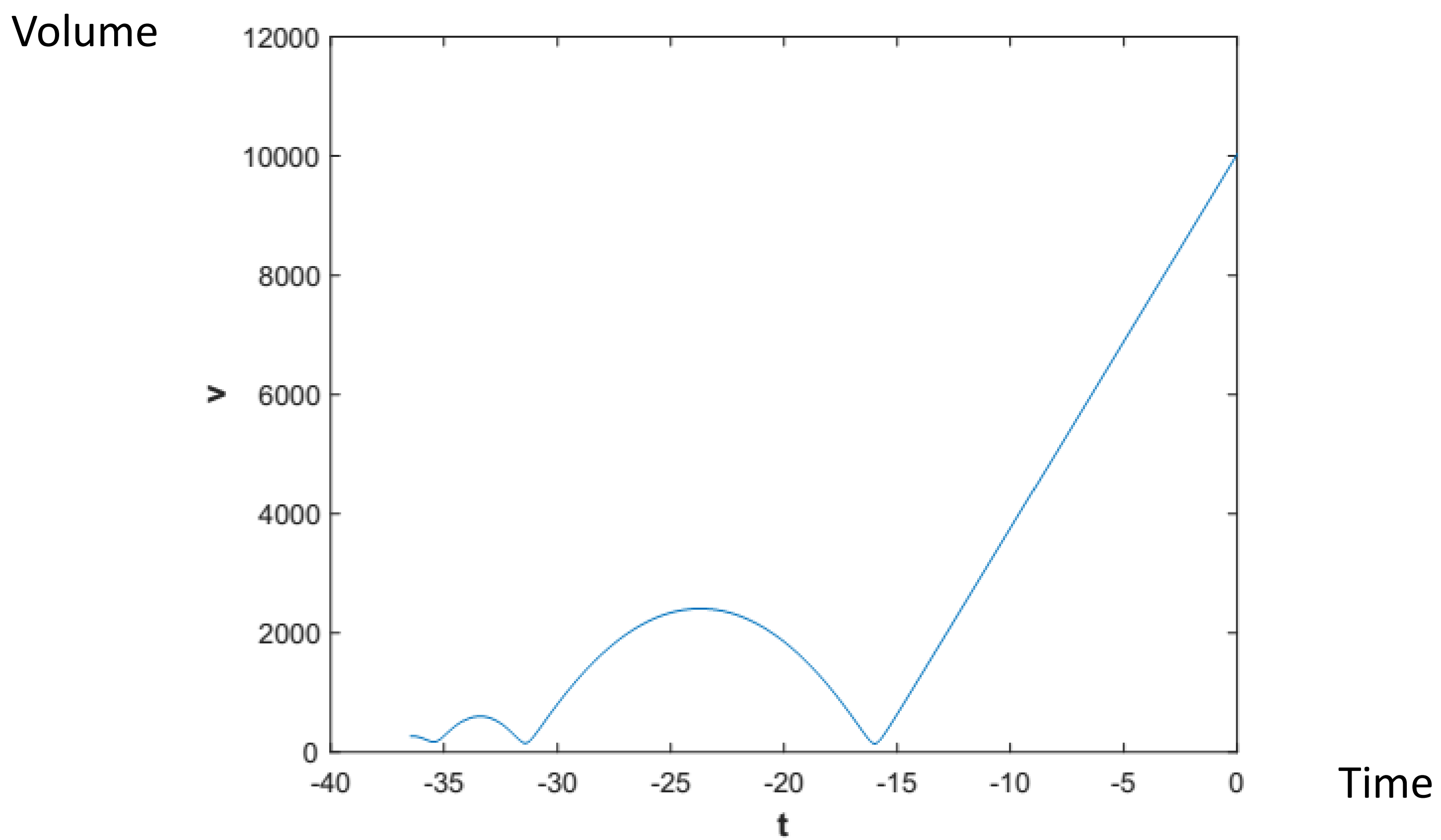
$$\bar{\rho}_{\mathrm{cr}} = -\frac{1}{\Delta}\,,$$

$$\Omega_{\mathrm{g}} = -\Delta\,\rho_{\mathrm{g}} = \frac{\rho_{\mathrm{g}}}{\bar{\rho}_{\mathrm{cr}}}\,,$$

$$\Omega_{\mathrm{m}} = \frac{\rho_{\mathrm{m}}}{\rho_{\mathrm{cr}}}\,.$$

$$\left(\frac{\dot{a}}{a}\right)^2 = \left(\frac{8\pi}{3}\rho_{\mathrm{m}} + \frac{\rho_{\mathrm{g}}}{\gamma^2}\right)(1-2\Omega_{\mathrm{g}})^{-1}\left(1 - \frac{\Omega_{\mathrm{m}} - \Omega_{\mathrm{g}}}{1-2\Omega_{\mathrm{g}}}\right)$$
$$\left(1 + \frac{2\Omega_{\mathrm{g}}}{b\sqrt{\Delta}}\cot(2b\sqrt{\Delta}) - 2\Omega_{\mathrm{g}}\right)^2.$$

The emergent Bouncing Universe



$$\Omega_m + \Omega_{geo} = 1$$ Min

$$\Omega_m = \Omega_{geo}$$ Max

Dynamical resolution of the singularity in the deep planckian regime

Work in progress

Computable Arena for LQG

Cosmology

- Evolution of the complete Hamiltonian *EA Botta Pawlowski*
- Study the Physical Hilbert space, Graph Changing Hamiltonian *EA Apadula*
- Link to LQC phenomenology: Power Spectrum *EA, Botta, Barrau, Martineau, Stagno*
- Everything can be extended to Bianchi: Evolution, Isotropization, role of new quantum corrections *EA Botta Stagno*

Black Holes

- Reduced Phase Space for Radial gauge in Ashtekar Variables *EA Pacilio Pranzetti*
- QRLG: Black Holes (Kinematics is ready) *EA Liberati Pacilio Pranzetti*

Matter

- Scalar field and U(1) gauge fields included: *EA, Bilski, Cianfrani, Dona, Marciano*
- Effective QFT: Non local ? *EA Letizia Liberati Pranzetti*